EPFL

Plasma II
L13: Plasma diagnostics 2

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Swiss Plasma Center

Outline

- Introduction
- Passive measurements
  - EM emission
    - Bremsstrahlung
    - Line emission
    - Synchrotron radiation
    - Cyclotron emission
  - Magnetic fields
- **Active measurements**
  - Langmuir probes
  - Interferometry
  - Thomson scattering
- **Appendix**

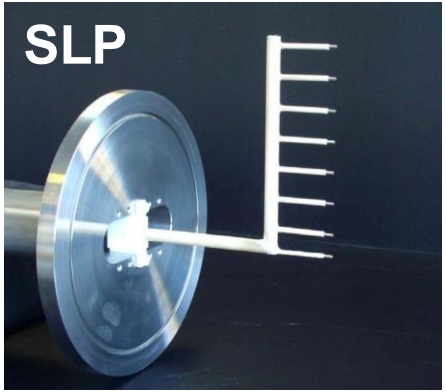
Plasma diagnostics 1

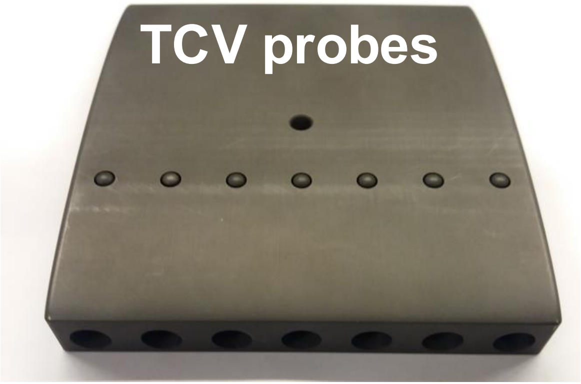
Plasma diagnostics 2

- I.H. Hutchinson, "*Principles of plasma diagnostics*", Second edition, Cambridge University Press

Langmuir probes

- One of the simplest and certainly one of the most widely used plasma diagnostics
  - Not necessarily easy to interpret



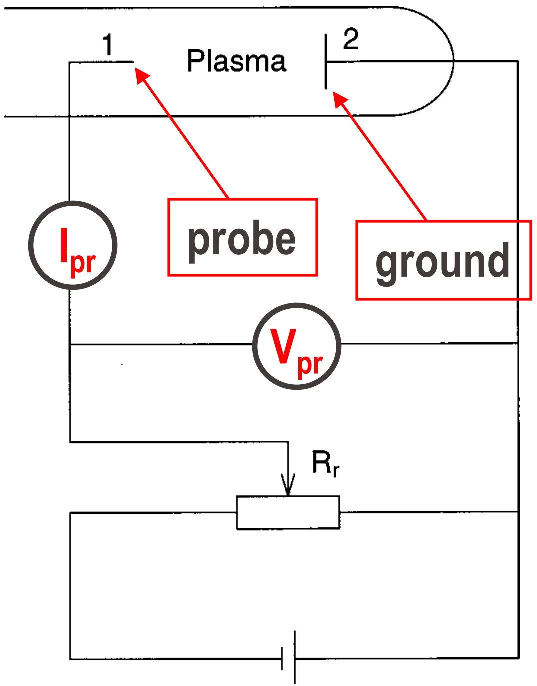


Irving Langmuir (1881-1957), American chemist, Physicist and engineer.
- Originator of the term ‘plasma’
- Nobel prize in chemistry 1932

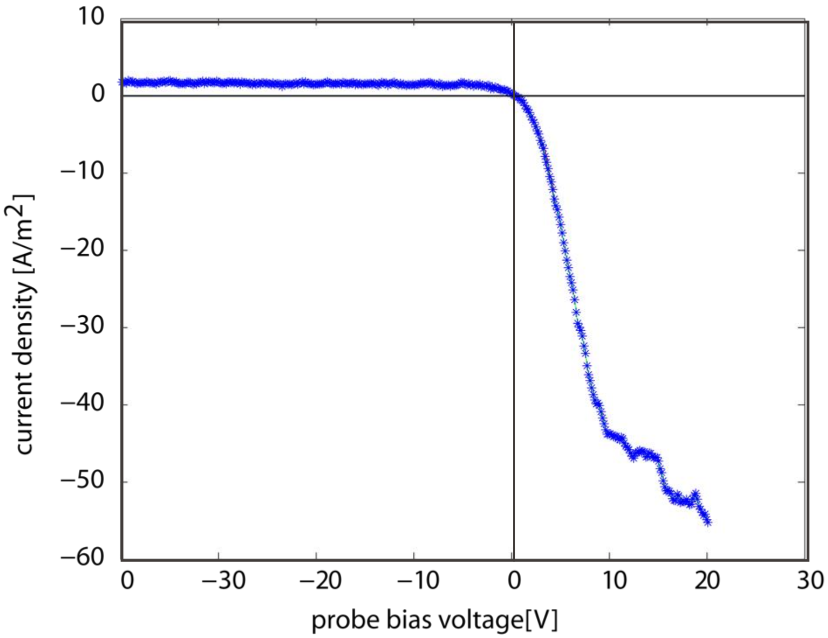
- Associated with Irving Langmuir who was one of the first to use electric probes to sense plasma fluxes

Langmuir probes measure a current as a function of an applied potential

- Circuit diagram



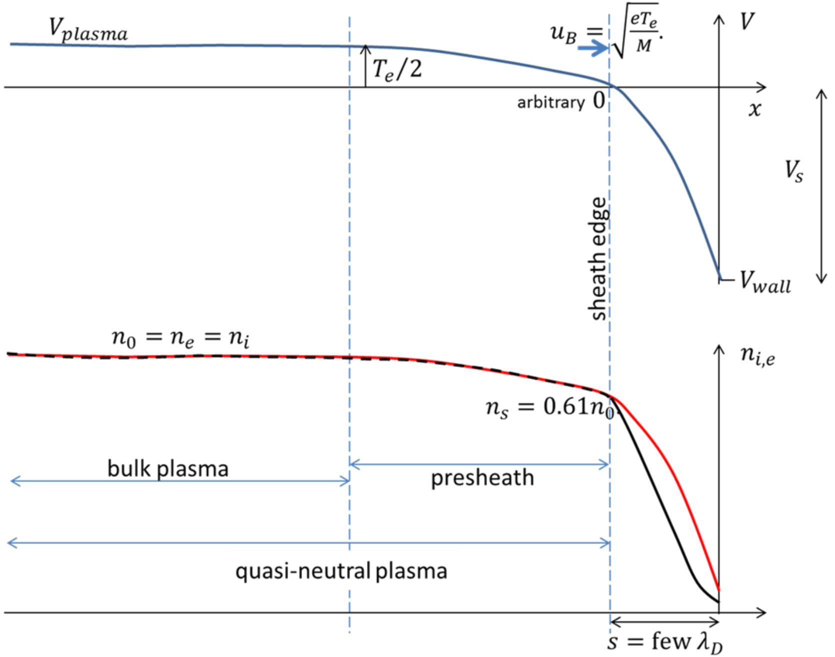
- Typical I-V curve



Reminder of L8

- Floating wall

$$V_{\mathrm{plasma}} = +k_{\mathrm{B}}T_e/2e$$



$$V_{\mathrm{sheath-entrance}} \overset{!}{=} 0$$

$$V_{\mathrm{wall}} = -\frac{k_{\mathrm{B}}T_e}{2e}\ln\left(\frac{M}{2\pi m_e}\right)$$

$$= V_{\mathrm{float}}$$

Langmuir probes
Theory

- Plasma sheath (L8)
  - Ion flux to the wall: $\Gamma_i = n_s u_B = n_s\sqrt{\frac{k_B T_e}{M}} = n_s \frac{\bar{v}_e}{4} \exp\left(\frac{eV_{\text{float}}}{k_B T_e}\right)$
  - Electron flux to the wall: $\Gamma_e = n_{e,\text{wall}} \frac{\bar{v}_e}{4} = n_s \frac{\bar{v}_e}{4} \exp\left(\frac{eV_{\text{wall}}}{k_B T_e}\right)$

Eq. that we used to determine V_{float} (potential of a floating wall)

General case: Allow V_{wall} to deviate from V_{float}

- Current density

$$j = e(\Gamma_i - \Gamma_e) = en_s \frac{\bar{v}_e}{4}\left[\exp\left(\frac{eV_{\text{float}}}{k_B T_e}\right) - \exp\left(\frac{eV_{\text{wall}}}{k_B T_e}\right)\right]$$

$$= \underbrace{en_s \frac{\bar{v}_e}{4} \exp\left(\frac{eV_{\text{float}}}{k_B T_e}\right)}_{n_s u_B}\left[1 - \exp\left(\frac{e(V_{\text{wall}} - V_{\text{float}})}{k_B T_e}\right)\right]$$

No current when V_{wall}=V_{float} (as expected)

Langmuir probes
Interpretation of I-V curve

Potential of surrounding wall – usually used as ground for the probe

- Current density

$$I_{\mathrm{pr}} = A_{\mathrm{pr}}\ j = \underbrace{A_{\mathrm{pr}} e n_s u_B}_{I_{\mathrm{sat}}} \left[1 - \exp\left(\frac{e(V_{\mathrm{wall}} - V_{\mathrm{float}})}{k_{\mathrm{B}} T_e}\right)\right]$$

Probe bias : negative bias only suppresses electron current, but cannot draw more ion current

Langmuir probes
Interpretation of I-V curve

- Current density

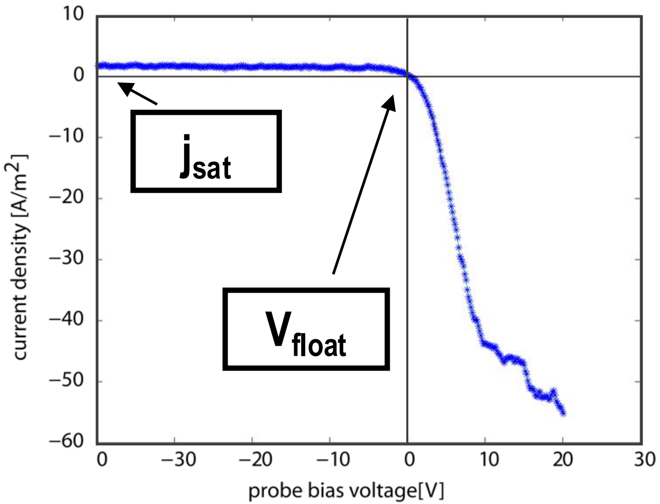
$$j_{\mathrm{pr}} = j_{\mathrm{sat}}\left[1 - \exp\left(\frac{e(V_{\mathrm{bias}} - V_{\mathrm{float}})}{k_{\mathrm{B}}T_e}\right)\right]$$

  - With ion saturation current

$$j_{\mathrm{sat}} = e n_s u_B$$

1. Particle flux to the wall

$$\Gamma_{\mathrm{wall}} = n_s u_B = j_{\mathrm{sat}}/e$$



Langmuir probes
Interpretation of I-V curve

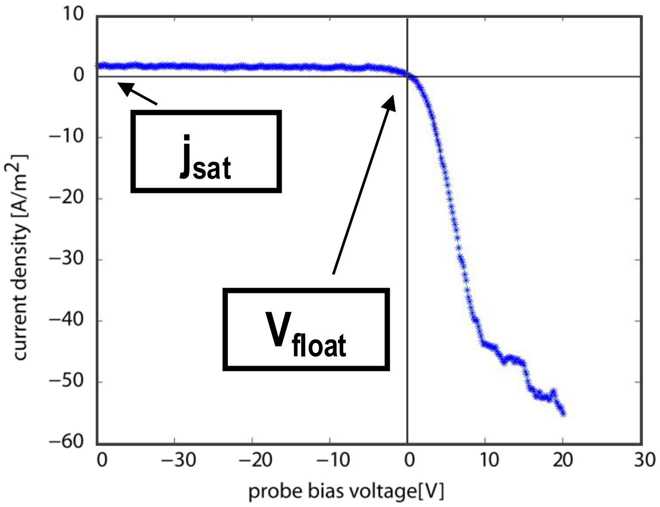
- Current density

$$j_{\mathrm{pr}} = j_{\mathrm{sat}} \left[1 - \exp\left(\frac{e(V_{\mathrm{bias}} - V_{\mathrm{float}})}{k_{\mathrm{B}} T_e} \right) \right]$$

  - With ion saturation current

$$j_{\mathrm{sat}} = e n_s u_B$$

2. Electron temperature T_e determined from logarithmic slope of $j_{\mathrm{pr}} = f(V_{\mathrm{bias}})$



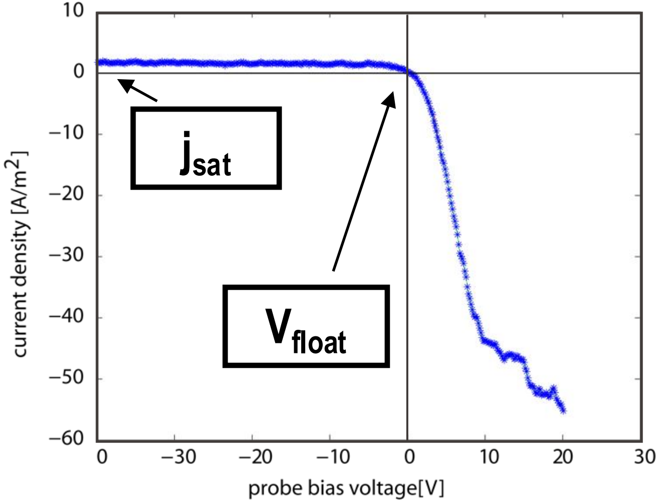
Langmuir probes
Interpretation of I-V curve

- Current density

$$j_{\mathrm{pr}} = j_{\mathrm{sat}}\left[1 - \exp\left(\frac{e(V_{\mathrm{bias}} - V_{\mathrm{float}})}{k_{\mathrm{B}}T_e}\right)\right]$$

  - With ion saturation current

$$j_{\mathrm{sat}} = e n_s u_B$$



3. Plasma density

$$n_0 = \frac{n_s}{0.61} = \frac{j_{\mathrm{sat}}}{0.61\,e}\sqrt{\frac{M}{k_{\mathrm{B}}T_e}}$$

$$= \frac{j_{\mathrm{sat}}}{0.61\,e}\sqrt{\frac{M}{k_{\mathrm{B}}T_e(1 + T_i/T_e)}}$$

if T_i >0

Langmuir probes
Interpretation of I-V curve

- Current density

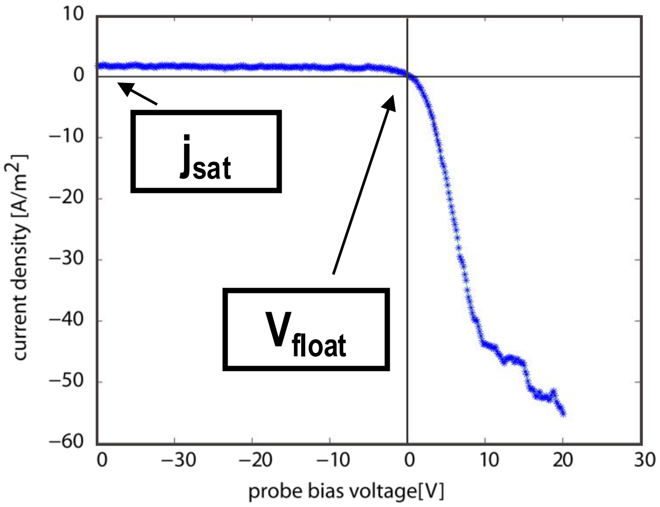
$$j_{\mathrm{pr}} = j_{\mathrm{sat}}\left[1 - \exp\left(\frac{e(V_{\mathrm{bias}} - V_{\mathrm{float}})}{k_{\mathrm{B}}T_e}\right)\right]$$

  - With ion saturation current

$$j_{\mathrm{sat}} = e n_s u_B$$

4. Plasma energy loss (evaluated at sheath entrance)

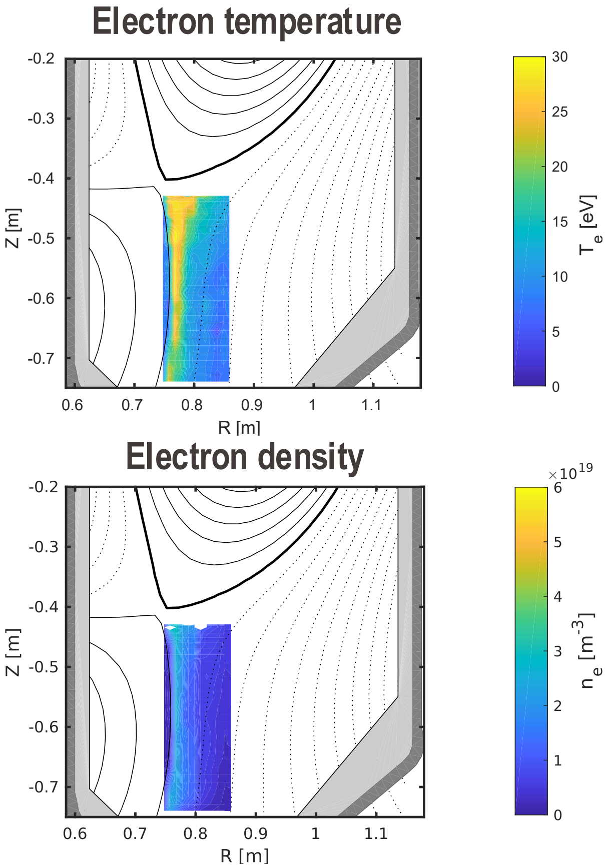
$$q_{\mathrm{se}} = \left[\frac{5}{2}k_{\mathrm{B}}T_i + \frac{1}{2}Mu_{\mathrm{B}}^2\right]\frac{j_{\mathrm{sat}}}{e} + \left[2k_{\mathrm{B}}T_e + e(|V_{\mathrm{sheath}}|)\right]\frac{j_{\mathrm{sat}}}{e} + E_{\mathrm{pot}}\frac{j_{\mathrm{sat}}}{e}$$



Langmuir probes
Examples

- Wall mounted
- Movable probes
  - Ex. TCV's reciprocating divertor probe arrays (RDPA)

[Courtesy of H. De Oliveira]



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  - **Interferometry**
  - Thomson scattering
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Plasma diagnostics 1

Plasma diagnostics 2

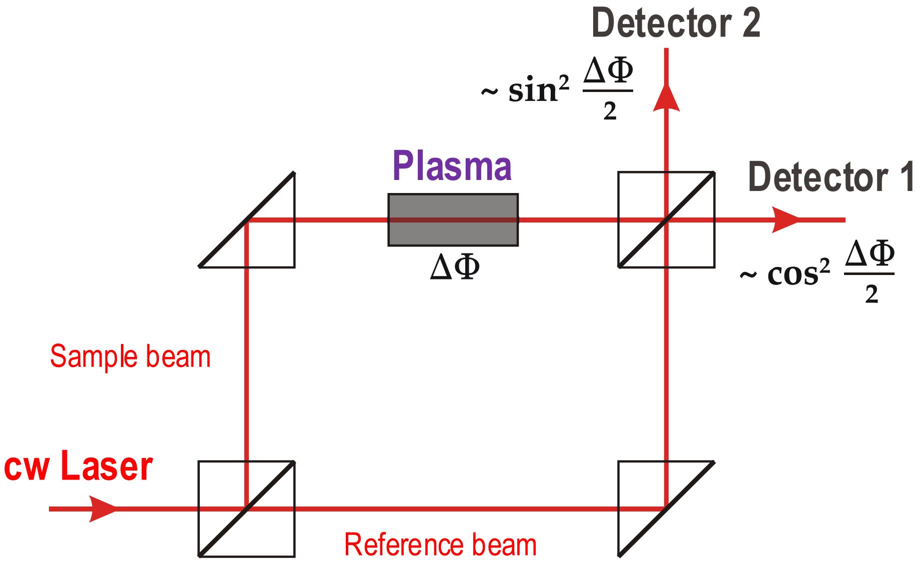
Interferometry theory

- Use propagation of light through a plasma to infer its density
  - Refractive index $N = \sqrt{1 - \frac{\omega_{p.e}^2}{\omega^2}}$ with electron plasma frequency $\omega_{p,e} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$

Interferometry

- Measure phase shift $\Delta\phi$ with a ***Mach-Zehnder interferometer***

- Split coherent light into sample and reference beams
  - Without phase shift beams interfere constructively at detector 1 and destructively at detector 2
  - Absolute phase shift known up to π → Need to keep track of multiples of π → count fringes
  - Ambiguity of phase change direction, when $\Delta\phi = 0, \pi, 2\pi, \ldots$



Source: Ansgar Hellwig, https://commons.wikimedia.org/w/index.php?curid=348005

Interferometry

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Polarimetry theory

- **Faraday rotation**: Right and left circularly polarised light travelling parallel to a magnetic field in a plasma have different refractive indices

$$n_{L,R} = \sqrt{1 - \frac{\omega_{p,e}^2}{\omega^2 \pm \omega\Omega_e}} \quad \text{with} \quad \Omega_e = \frac{eB}{m_e} \quad \text{and} \quad \omega_{p,e} = \sqrt{\frac{n_e e^2}{m_e \varepsilon_0}}$$

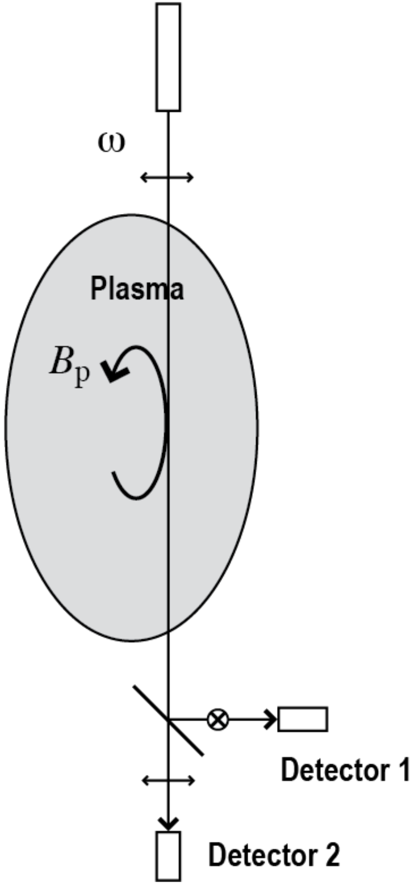
The polarisation angle θ of linearly polarized light rotates. The rotation angle is proportional to

$$\Delta\theta_{\mathrm{F}} \propto \lambda^2 \int n_e \mathbf{B} \cdot \mathbf{dl}$$

- Faraday rotation in typical fusion plasmas is small $\Delta\theta_{\mathrm{F}} \ll \pi$ ➔ requires sensitive detection technique

Polarimetry

- In typical fusion plasmas Faraday rotation is measured using light in the far-infrared (100 - 400μm)
- Simple polarimeter scheme:
  - Linearly polarized light is passed through a plasma
  - Faraday rotation changes the polarisation angle
  - A polarisation sensitive beam splitter separates orthogonal components
  - Ratio of amplitudes at detector 1 and 2 yields the Faraday rotation
- Various other detection techniques have been developed



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Plasma diagnostics 1

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Thomson scattering theory

- Use scattering of laser light propagating through a plasma to infer electron temperature
- Scattering of light on free charges in the classical limit ($h\nu \ll mc^2$) is referred to as Thomson scattering
  - Charges (primarily electrons) accelerating in the time-varying electric field of an incoming e-m wave emit e-m radiation (scattered light)
  - Cross section for scattering on free-electron $\sigma_{\mathrm{TS}} = \frac{8}{3}\pi r_c^2$ with $r_c = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}$ being the classical electron radius is small (**see L9**)
  - ➢ Only a small fraction ($\sim 10^{-11}$) of the laser light is scattered

Thomson scattering theory

- Equation of motion for an electron

$$m_e \frac{d\bar{v}}{dt} = -e\bar{E}_i$$

- Emits an e-m wave

$$\bar{E}_s = -\frac{e}{4\pi\varepsilon_0}\left[\frac{1}{c^2R}\hat{s} \times \left(\hat{s} \times \frac{d\bar{v}}{dt}\right)\right] = \frac{e}{4\pi\varepsilon_0 m_e c^2 R}[\hat{s} \times (\hat{s} \times \bar{E}_i)] = \frac{r_c}{R}[\hat{s} \times (\hat{s} \times \bar{E}_i)]$$

- Emitted power depends on scattering direction

$$\frac{dP}{d\Omega_s} = R^2 c\varepsilon_0 |\bar{E}_s|^2 = r_c^2 c\varepsilon_0 \sin^2\phi\, |\bar{E}_i|^2$$

➢ Polarise incident laser light perpendicular to observation direction

Thomson scattering spectrum

- Spectrum for a Maxwellian electron velocity distribution

$$\frac{d^2P}{d\omega d\Omega_s} = r_c^2 n_e P_i L \sin^2\phi \, S(\bar{k},\omega)$$

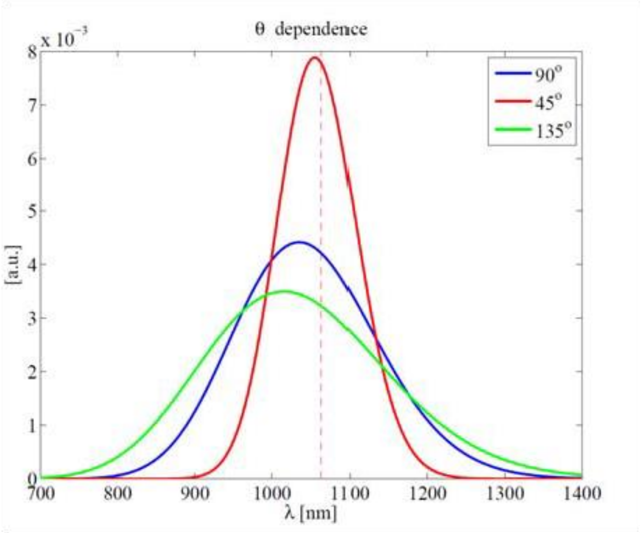
  - With spectral form factor S

- Thermal motion of electrons broaden spectrum of scattered light

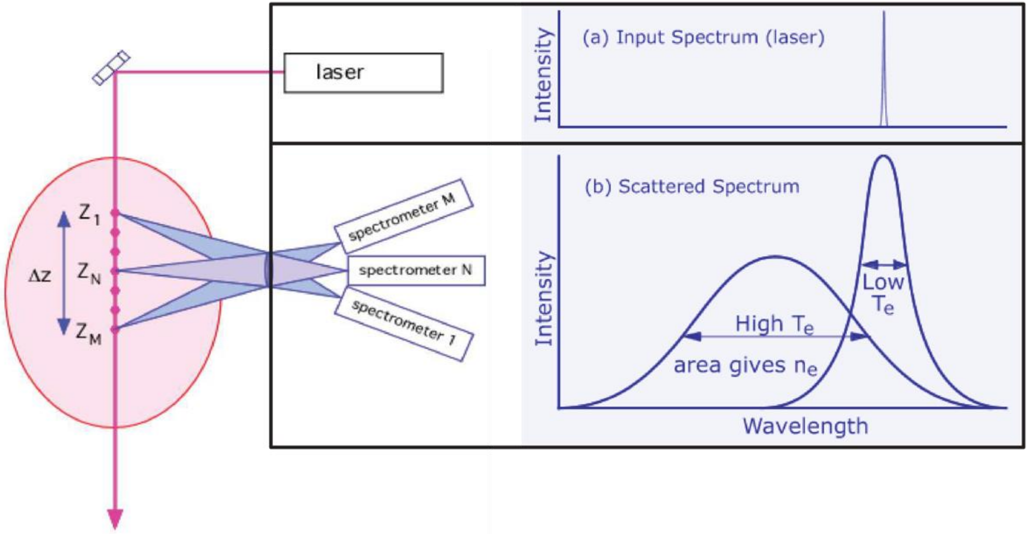
➢ Spectrum yields electron temperature

$$\Delta\lambda_{\mathrm{FWHM}} = 4\varepsilon\lambda_0 \sin\frac{\theta}{2}\sqrt{\frac{2k_{\mathrm{B}}T_e \ln 2}{m_e c^2}}$$

  - Depends on observation angle θ



Thomson scattering setup



Beam dump

- Broadening yields electron temperature
- Intensity yields electron density

$$n_e \propto \int P(\bar{k}, \omega) d\omega$$

- Low scattering cross section requires high power laser → pulsed operation
- TS diagnostic practically non-perturbative!

The end

Appendix

- Errors

Errors

- **Systematic errors**
  - Consistent and reproducible (e.g. calibration error)
- Cannot be revealed through repeated measurements

➢ Compare different measurements (technique or instrument) of same parameter

➢ Estimate from knowledge of technique

- **Statistical errors**
  - Random (e.g. fluctuations in electronics)

➢ Can be revealed by repeated measurements

- **Human errors**
  - Random (e.g. noting the wrong number)

➢ Care and self-correction

[I.H. Hutchinson, "Principles of plasma diagnostics", Appendix 2]

Accuracy vs. Precision

- Accuracy – deviation from reality
- Determined by all errors

- Precision – reproducibility
- Determined by statistical errors

[I.H. Hutchinson, “Principles of plasma diagnostics”, Appendix 2]

Statistical error

- Set of N repeated measurements of same parameter – *sample*
  - Mean value of sample

$$\mu_N = \frac{1}{N}\sum_{i=1}^{N} x_i$$

  - Standard deviation of sample

$$\sigma_N = \left[\frac{1}{(N-1)}\sum_{i=1}^{N}(x_i - \mu_N)^2\right]^{1/2}$$

- Increasing the sample size will lead to constant values μ_N and σ_N – characteristic of the random distribution of the parameters
  - μ : Associated with 'real' value of parameter
  - σ : Statistical uncertainty of each measurement x_i

[I.H. Hutchinson, "Principles of plasma diagnostics", Appendix 2]

Reduce uncertainty of estimate

- If σ is the standard deviation of measurements x_i, μ_N becomes a random variable described by a Gaussian probability distribution with a standard deviation $\sigma/\sqrt{N}$

➢ Standard deviation (uncertainty!) of sample mean decreases with number of measurements in each set -> ***standard error***

$$s_N = \frac{\sigma_N}{\sqrt{N}} = \left[\frac{1}{N(N-1)}\sum_{i=1}^{N}(x_i - \mu_N)^2\right]^{1/2}$$

- Increase accuracy with more measurement until systematic error dominates – further increase only increases precision, but not accuracy

[I.H. Hutchinson, "Principles of plasma diagnostics", Appendix 2]

Literature

Textbooks

- J. Wesson, "Tokamaks", Third edition, Oxford Science Publications – Chapter 10 ← *Good overview*
- I.H. Hutchinson, "Principles of plasma diagnostics", Second edition, Cambridge University Press ← *Very detailed description based on first principles, goes well beyond course content*